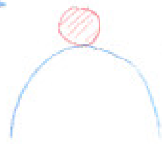
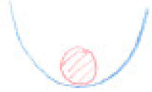
LYAPONOV STABILITY

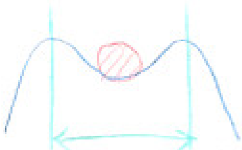
terms:



unstable equilibrium point



stable equilibrium point.

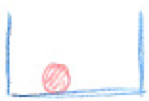


region of stability

local equilibrium point

a global equilibrium point.

* Stability → local, global
* Asymptoptic Stability → local, global
* Exponential Stability → local, global.



Stability; bounded.

Asymptoptic Stability; returns to the equilibrium point.

Exponential Stability ; returns to the equilibrium point exponentially.

Equilibrium Point

$$\dot{x} = f(x) \qquad x \in \mathbb{R}^n \quad (1)$$

The equilibrium point of system (1) are given by $f(x)=0$ let x^* be the equilibrium point of 1

let

error $\boxed{e = x - x^*}$

$$\dot{e} = \dot{x} - \dot{x}^* = f(x) - \cancel{f(x^*)}^{0} = f(e + x^*)$$

for this system the equlibrium point is $e=0$

LYAPUNOV FIRST METHOD

$\dot{x} = f(x)$ 1

$x = 0$ is equlibrium point for 1

let $\dot{x} = Ax$ 2 be the linear aprox of 1 around equlibrium point $x=0$.

$$f(x) = f(0) + (x-0)\left(\frac{\partial f}{\partial x}\right)_{x=0} + \dots$$

$$f(x) \cong \boxed{\left.\frac{\partial f}{\partial x}\right|_{x=0}} x$$

Jacobian.

A

* If A is stable $\Rightarrow$ then equ. pt. $x=0$ is exponential stable. 1
* If A is unstable $\Rightarrow$ the equ. pt. $x=0$ is unstable for system. 1
* If A has eigen values on the imaginary axis, we cannot conclude about the stability of the equ. pt. for 1

LYAPUNOV SECOND (DIRECT) METHOD

$V(x) : \mathbb{R}^n \to \mathbb{R}_+$

$x \in \mathbb{R}^n$; $V(x) \in \mathbb{R}_+$

$V(x)$ is positive definite if $V(x) > 0$ $\forall x \neq 0$
$V(0) = 0$

EX:

$$V(x) = x^T P x \qquad x \in \mathbb{R}^n$$

$$P = P^T$$

then P is symmetrical positive definite.

$$V(x) = x^T x \quad : ?$$

we can also have negative definite if

$$V(x) < 0 \; ; \; V(0) = 0 \; ; \; \forall x \neq 0$$

like wise

* positive semi definite

$$V(x) \geq 0$$

* negative semi definite

$$V(x) \leq 0$$

THEOREM: if there exist a positive definite function $V : \mathbb{R}^n \to \mathbb{R}_+$ is differentiable with respect to x and

negative semi definite

$$\frac{\partial}{\partial t} V(x) = \frac{\partial V}{\partial x} \dot{x} = \frac{\partial V}{\partial x} f(x) \leq 0$$

then the equi. pt $x = 0$ for system ① is stable and $V(x)$ is a LYapunov function.

THEOREM: there exists a positive definite function $V: \mathbb{R}^n \to \mathbb{R}_+$ is differentiable with respect to x and if

negative definite

$$\frac{\partial}{\partial t} V = \frac{\partial V}{\partial x} \dot{x} = \frac{\partial V}{\partial x} f(x) < 0$$

then the equ. pt. $x=0$ is assymtoptically stable and $V(x)$ is a Lyapunov function.

EX:

$$\dot{x} = x^2 + u \qquad \begin{matrix} x \in \mathbb{R} \\ u \in \mathbb{R} \end{matrix}$$

design u such that the equ point $x=0$ is globally assumptopically stable.

SOL:

let $V(x) = \frac{1}{2} x^2$

$$\dot{V} = \frac{\partial V}{\partial x} \dot{x} = x \dot{x} = x[x^2 + u]$$

taking $u = -x^2 - kx$

$$\dot{V} = -kx^2 < 0 \quad \text{(negative definite)}$$

$\therefore$ the equ. pt. $x=0$ is GAS (GES)